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Estimating correlation and covariance matrices by weighting of market similarity

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We discuss a weighted estimation of correlation and covariance matrices from historical financial data. To this end, we introduce a weighting scheme that accounts for the similarity of previous market conditions to the present situation. The resulting estimators are less biased and show lower variance than either unweighted or exponentially weighted estimators. The weighting scheme is based on a similarity measure that compares the current correlation structure of the market to the structures at past times. Similarity is then measured by the matrix 2-norm of the difference of probe correlation matrices estimated for two different points in time. The method is validated in a simulation study and tested empirically in the context of mean–variance portfolio optimization. In the latter case we find an enhanced realized portfolio return as well as a reduced portfolio risk compared with alternative approaches based on different strategies and estimators.

Keywords: Portfolio optimization; Correlation structures; Time series analysis; Market dynamics

JEL Classification: C1, C6, C13, C61, D4, G1, G11

1. Introduction

Good estimates of the correlation and covariance matrices of financial returns are central for a wide range of applications such as risk management, option pricing, hedging and capital allocation. For example, in risk management applications, they directly affect the calculation of the value at risk or the expected shortfall. In the context of capital allocation, the correlation structure is the key in the classical portfolio optimization problem, as shown by the seminal work of Markowitz (1952).

Generally, the quality of the estimated matrices increases with the length of the time series, i.e. the amount of data used. For small datasets the matrices have a large variance and may even be singular or indefinite. In a financial context, however, using long time series results in biased estimates of the correlation structure, since the dependence of asset returns is not constant over time (see, e.g. King and Wadhwani 1990 for an early review). The problem is that standard estimators equally weight all parts of the dataset. By consequence, out-of-date and improper information greatly affects the estimates. This paper tackles this problem by introducing a new weighted estimator of the correlation or covariance matrix. This estimator makes use of enough data to adequately limit its variance but, in order to minimize its bias, focuses only on parts of the data where the market is in similar market conditions, i.e. where it exhibits the same correlation structure.

To reduce the effects of time-changing structures, common approaches in the literature choose time intervals in which the structures are approximately constant. Examples of such approaches are exponentially weighted estimators like the RiskMetrics estimators (see, e.g. Longerstaey and Spencer 1996) or the estimators discussed by Lee and Stevenson (2003). Since these estimators only use a small part of the data, they show a large variance. Moreover, whenever the number of effectively used observations is not large compared with the dimensions of the time series, estimated correlation and covariance matrices may be regarded as completely random. Laloux et al. (1999) showed in an empirical study that, in such cases, 94% of the spectrum of estimated correlation matrices equals the spectrum of random matrices and only their largest eigenvalues may be estimated.
adequately. Solutions to this problem involve reducing the dimensionality of the problem by imposing some structure on the correlations, e.g. by using factor models or shrinkage estimators as in Ledoit and Wolf (2004) or by noise-reduction techniques, e.g. Random Matrix Filtering (Plerou et al. 2002) or Power Mapping (Schäfer et al. 2010).

In addition to the non-parametric estimators mentioned above, there are approaches such as the DCC model of Engle (see McNeil et al. 2005), which capture the time-changing structure directly. They use conditional dynamic models of the correlation matrices going back to the work of Bollerslev (1986). A short overview may be found in McNeil et al. (2005) or Andersen et al. (2007). These approaches, however, are parametric approaches, i.e. they assume parametric models for the dynamics and face model risk. The decision as to which of the numerous variations of the models in the literature fits the data best can only be made ex-post and only after comparing results from various models. The focus of our paper is therefore on how to improve non-parametric estimation with respect to bias and variance of the estimators.

With the availability of intra-day high-frequency financial data, it was expected that finer sampled data would effectively enlarge the datasets and improve estimates of parameters. However, when return data are observed at shorter time intervals, they are contaminated by market microstructure effects. These effects influence estimators and induce bias and noise (see, e.g., Bandi et al. 2008 for a recent discussion). Possible reasons for this include asynchrony and decimation effects (see, e.g. Münix et al. 2010a, b, c).

Since the amount of data for the estimation may only be increased by either considering a longer time period or by sampling at higher frequencies, the mentioned properties of financial time series limit the amount of usable data. Longer time intervals bias the estimators due to the time-changing nature of the matrices. Higher frequencies intensify the effects of the market microstructure on the estimators.

In this paper, we circumvent these limits. We propose to enlarge the amount of usable data by adaptively including different parts of the time series with similar correlation structures in the estimator. We therefore introduce a similarity measure that measures the degree of similarity between days of the time series based on probe correlation estimates. We demonstrate the application of the measure by assessing the similarities on stock returns from the S&P 500 index. The measure reliably detects regime changes in the data as well as the particular market situation during the financial crisis in 2008. The similarity measure enables us to construct a weighting scheme for correlation or covariance estimators that attaches large weights to similar parts of the data and suppresses distortions. In a simulation study, we demonstrate that these similarity weighted estimators show smaller bias and variance than unweighted or exponentially weighted estimators. The results hold for constant as well as for dynamic correlation structures in the data. In a real data application we apply our estimator to covariance estimation in the context of mean–variance portfolio optimization. We use time series of stocks from the S&P 500 index and randomly choose stocks to build up portfolios. We show that optimal portfolios that are based on the similarity weighted covariance estimator outperform alternative approaches with respect to realized volatility and realized return.

The paper is organized as follows. Section 2 introduces the measure of similarity. In section 3, a similarity-based weighting scheme for estimators of correlation or covariance is constructed. Section 4 contains a simulation study analysing the variance and bias of the resulting estimators. In section 5 we empirically apply the estimators in the context of mean–variance portfolio optimization. Section 6 concludes.

2. Measuring market similarity

We measure the degree of similarity $\xi$ in the market’s correlation structure by the norm of the difference of the correlation matrices $C(t_1)$ and $C(t_2)$ at times $t_1$ and $t_2$.

$$\xi^L(t_1, t_2) = \|C^L(t_1) - C^L(t_2)\|_2.$$  (1)

The correlation matrices $C^L(t_1)$ and $C^L(t_2)$ are estimated on a backward-looking moving window of length $L$. The window length $L$ is indicated by a superscript, i.e. $\xi^L$. Here $\|C\|_2$ represents the induced matrix 2-norm (or Euclidean norm) of the real-valued matrix $C$, which, in the case where $C$ is symmetric, corresponds to the absolute value of its largest eigenvalue. To reduce the impact of outliers in the data, the estimates are based on Spearman’s rank correlation instead of Pearson’s product moment correlation, as this estimator is more robust to non-normal distributions. Since the estimates should be unbiased for time-varying correlations, the use of small window lengths is recommended. As discussed by Laloux et al. (1999), this results in noisy estimates of the matrices and only the largest eigenvalues of the matrices are adequately estimated. However, the similarity measure (1) is based on the 2-norm and thus depends only on this largest eigenvalue, which can be estimated even for small values of $L$.

Figure 1 illustrates the evolution of the similarity measure $\xi^{50}$ for the example of the 471 assets that were continuously listed in the S&P 500 index between August 2005 and January 2010. The similarity measure is evaluated for every day within this period and depicted as a matrix. Each point of the graphic reflects the degree of similarity between the days at its coordinates. The axes represent time, therefore the evolution of the market related to a specific point in time is given by the upright (or vertical) intersection through this point. Darker regions on this intersection are less similar and brighter regions more similar to the situation at the specific point in time. In this illustration, the financial crisis is clearly visible by the dark-shaded area from October 2008 to March 2009. The correlation structure in this period is completely different from any period before. After this period we find the market stabilizing: the correlation...
correlation increases, indicating the new correlation regime. The sharp transition in February 2007 was induced by a large overall price drop of the stocks on the S&P 500. This originated in drastic events on the Chinese stock market.†

3. Similarity weighted estimators

The similarity measure $\xi^L$ may serve as a weighting scheme for estimators of correlation or covariance matrices. With respect to the reference point $t_0$ the scheme ascribes high weights to periods where the market behaved in a similar manner. On the other hand, the periods in which the market behaved very differently are suppressed. Therefore, we consider the adapted similarity measure

$$\tilde{\xi}^L(t, t_0) = 1 - \frac{\xi^L(t, t_0)}{2(K - 1)}, \quad t \in [t_0 - T, t_0],$$

where $T$ is the total number of considered time steps, i.e. the length of the time series. The factor $K$ refers to the number of assets to include. $2(K - 1)$ represents the theoretical maximum possible value for $\xi$, i.e. the greatest possible dissimilarity. This is easily checked by calculating the difference of the correlation matrices for the fully correlated case and the fully anticorrelated case. The value of all non-diagonal entries of the resulting matrix is 2, while the diagonal entries are zero. Therefore, the largest eigenvalue of this matrix is $2(K - 1)$.

We note that the probe matrices $\hat{C}^L$ in equation (1) are estimated with window length $L$. Therefore, within the time-span $[t_0 - L, t_0]$, they share identical values with the probe matrix at $t = t_0$. $\tilde{\xi}^L(t, t_0)$ is then dominated by the number of identical values and not by the estimated similarity. In particular, the identical values can affect the similarity measure by orders of magnitude. Therefore, the similarity measure is not reliable within this region and is set to the maximum value of the other time-spans, resulting in a corrected measure

$$\tilde{\xi}^L(t, t_0) = \begin{cases} \max(\tilde{\xi}^L(t < t_0 - L, t_0)), & t \in [t_0 - L, t_0], \\ \tilde{\xi}^L(t, t_0), & t \in [t_0 - T, t_0 - L]. \end{cases}$$

By choosing the maximum similarity value, we assume a high similarity of the most recent data, which should be reasonable in most cases. However, the actual choice of similarity for the region $t \in [t_0 - L, t_0]$ only has a small impact on the overall weighting, since usually $L \ll T$ as long as deterministic effects in $\tilde{\xi}^L$ are excluded. A normalized weighting scheme for the estimation of the correlation or covariance matrix $C(t_0)$ or $\Sigma(t_0)$ at time $t = t_0$ is then given by

$$w(t, t_0, L) = \tilde{\xi}^L(t, t_0) \left( \frac{\sum_{t' = t_0-T}^{t_0} \tilde{\xi}^L(t', t_0)}{\sum_{t' = t_0-T}^{t_0} \tilde{\xi}^L(t', t_0)} \right),$$

†See, for example, the cover story of Bloomberg Businessweek, March 12, 2007: What the market is telling us.
resulting in the weighted estimators

\[
\tilde{C}(t_0) = \sum_{t=t_0-T}^{t_0} w(t, t_0, L) \tilde{C}^L(t), \quad (5)
\]

\[
\tilde{\Sigma}(t_0) = \sum_{t=t_0-T}^{t_0} w(t, t_0, L) \tilde{\Sigma}^L(t). \quad (6)
\]

The superscript \( L \) again denotes the respective window length of the estimators.

For both estimators the weights are based on probe correlation estimates. In the case of the covariance estimator it may seem more appropriate to use a similarity measure based on probe covariance estimates. Then, however, a normalization of the measure as in equation (2) would be problematic since covariances are, in contrast to correlations, unbounded. Hence, a proper normalization is required, e.g. by normalizing by the standard deviation. The covariance matrix of such standardized data, however, corresponds to its correlation matrix, which leads to the above weights.

For large \( T \) and time series with a dynamic correlation structure, the weighting scheme should be restricted to the \( s \) largest values of \( w \). This leads to a complete suppression of dissimilar parts of the data. Let \( w_{(i)} \) denote the \( i \)th largest value of \( w \). The restricted scheme \( w_s \) is then given by

\[
w_s(t, t_0, L) = |w - w_{(i)}|_+ \sum_{t=m-t}^{t} |w - w_{(j)}|_+, \quad (7)
\]

with

\[
|w - w_{(i)}|_+ = \max(w(t', t_0, L) - w_{(i)}, 0). \quad (8)
\]

The unbiasedness of the estimators (6) in time series, where the underlying correlation matrix is constant, is easily checked. However, due to fluctuations of the weights \( w \), their variances are expected to be slightly larger than for a constant non-adaptive weighting scheme \( w = 1/T \). These effects are explored in the simulation study in the next section.

4. Simulation study

The study presented here attempts to validate the estimators introduced in the last section. We estimate the correlation and compare it with standard estimators with respect to bias and variance. Bias and variance of the estimators are calculated as the deviation of the sample mean from the theoretical value (bias) and the sample variance of the estimates of 400 repetitions of the study, respectively. The study consists of three scenarios of normally distributed daily returns of 16 assets. The scenarios are constructed similarly to the testing environments of Pafka and Kondor (2004). The first scenario is equicorrelation with the parameter \( \rho = 0.7 \). This means that all pairwise correlations of the correlation matrix \( \rho_{ij} \) of the 16 asset returns are equal to \( \rho = 0.7 \) for \( i \neq j \).

In the second and third scenarios, the market consists of two equicorrelated branches, the first eight assets with parameter \( \rho_1 \) and the second eight assets with \( \rho_2 \). The assets of two different branches are equicorrelated with the parameter \( \rho_2 = 0.2 \). The equicorrelation parameters of the branches change over time, i.e. \( \rho_1 = \rho_1(t) \) and \( \rho_3 = \rho_3(t) \). In the second scenario, the market switches deterministically in turn between three different regimes. Each regime lasts 100 trading days. In regime 1, the branches are equicorrelated with parameters \( \rho_1 = 0.7 \) and \( \rho_3 = 0.3 \). In the second regime, these parameters are both equal to 0.5, and in the third regime they are 0.3 and 0.7, respectively. In the third scenario, the parameters \( \rho_1(t) \) and \( \rho_3(t) \) change sinusoidally with the trading days \( t \) according to

\[
\rho_1(t) = 0.4 + 0.3 \sin \left( \frac{t}{600} \frac{2\pi}{2} \right), \quad (9)
\]

\[
\rho_3(t) = 0.4 + 0.3 \sin \left( \frac{t - 300}{600} \frac{2\pi}{2} \right). \quad (10)
\]

Figure 3 depicts the theoretical similarity matrices of the second and third scenario for the first 1000 trading days. The discrete regimes of the second scenario are clearly visible while the similarity matrix of the third scenario shows no sudden changes.
The similarity weighted estimator is compared with the benchmark estimators. The first benchmark is the standard Pearson correlation estimator based on the last 300 returns. As the second benchmark, we use the RiskMetrics exponentially weighted correlation estimator. The estimator weights the \( j \)-th recent return with weight \( w_j \). The weights are chosen according to

\[
    w_j = \left( \frac{1 - \lambda^T}{1 - \lambda} \right)^{-1} \sum_{t=1}^{T} \lambda^{T-1} \approx (1 - \lambda) \sum_{t=1}^{T} \lambda^{T-1},
\]

with \( \lambda = 0.94 \) as suggested by Longstaey and Spencer (1996).

We estimate the correlation matrix in all three scenarios for the days \( t = 1000, t = 2500 \) and \( t = 5000 \). The correlation matrix of the first scenario has only one parameter \( \rho \), while the matrices of the second and third scenarios have three parameters \( \rho_1, \rho_2 \) and \( \rho_3 \). We estimate these parameters by averaging the corresponding entries of the estimated matrices. To estimate the mean and variance of these estimators, each simulation is independently repeated 400 times. The results are presented in tables 1–3, which show the means and sample standard deviations of the parameters of interest over the 400 repetitions for the three estimators.

Table 1 shows the results of scenario 1. The parameter \( \rho \) is estimated adequately in all cases, which confirms that all estimators are unbiased in this setup. As expected, the standard estimator has the lowest variance. It uses a constant weighting scheme. This is known to be optimal when the underlying correlation is constant. In this setup, the adaptive weighting scheme of

| Day 1000 | \( \rho = 0.7 \) | 0.6974 | 0.0364 | 0.6979 | 0.0296 | 0.6947 | 0.0736 |
| Day 2500 | \( \rho = 0.7 \) | 0.6991 | 0.0403 | 0.7002 | 0.0288 | 0.6977 | 0.0729 |
| Day 5000 | \( \rho = 0.7 \) | 0.6973 | 0.0429 | 0.7004 | 0.0296 | 0.7022 | 0.0718 |

Table 2. Simulation results for scenario 2, the scenario of discrete regimes in the correlation structure. Shown are the results for the similarity weighted, the unweighted and the exponentially weighted estimator.

| Day 1000 | \( \rho_1 = 0.7 \), \( \rho_2 = 0.2 \), \( \rho_3 = 0.3 \) | 0.6605 | 0.0339 | 0.4992 | 0.0448 | 0.6911 | 0.0759 |
| Day 2500 | \( \rho_1 = 0.7 \), \( \rho_2 = 0.2 \), \( \rho_3 = 0.3 \) | 0.6792 | 0.0341 | 0.4985 | 0.0456 | 0.6883 | 0.0734 |
| Day 5000 | \( \rho_1 = 0.5 \), \( \rho_2 = 0.2 \), \( \rho_3 = 0.3 \) | 0.4992 | 0.0492 | 0.4972 | 0.0448 | 0.5010 | 0.1072 |

Table 3. Simulation results for scenario 3, the scenario with sinusoidally changing correlation structure. Shown are the results for the similarity weighted, the unweighted and the exponentially weighted estimator.

| Day 1000 | \( \rho_1 = 0.1402 \), \( \rho_2 = 0.2 \), \( \rho_3 = 0.6598 \) | 0.2144 | 0.0548 | 0.4941 | 0.0457 | 0.1927 | 0.1391 |
| Day 2500 | \( \rho_1 = 0.6598 \), \( \rho_2 = 0.2 \), \( \rho_3 = 0.1402 \) | 0.6121 | 0.0478 | 0.3062 | 0.0540 | 0.6029 | 0.0891 |
| Day 5000 | \( \rho_1 = 0.6598 \), \( \rho_2 = 0.2 \), \( \rho_3 = 0.1402 \) | 0.6962 | 0.0361 | 0.6555 | 0.0457 | 0.6767 | 0.0804 |

The similarity weighted estimator is compared with the benchmark estimators. The first benchmark is the standard Pearson correlation estimator based on the last 300 returns. As the second benchmark, we use the RiskMetrics exponentially weighted correlation estimator. The estimator weights the \( j \)-th recent return with weight \( w_j \). The weights are chosen according to

\[
    w_j = \left( \frac{1 - \lambda^T}{1 - \lambda} \right)^{-1} \sum_{t=1}^{T} \lambda^{T-1} \approx (1 - \lambda) \sum_{t=1}^{T} \lambda^{T-1},
\]

with \( \lambda = 0.94 \) as suggested by Longstaey and Spencer (1996).
the similarity weighted estimator should also equally weight all observations. However, due to stochastic fluctuations, the weights vary. Therefore, the variance of the estimator is slightly larger than the variance of the standard estimator. The exponentially weighted scheme has the highest variance as it heavily weights the most recent observations. This results in an unbalanced weighting scheme that is not optimal in this scenario.

The results of scenario 2 are shown in table 2. The standard estimator is highly biased since its weighting scheme weights data from all three regimes equally. Unlike the standard estimator, the exponential estimator weights the most recent observations most and therefore seems unbiased. Again, its variance is the largest of the three considered estimators. The similarity weighted estimator shows variances comparable to the variance of the unweighted estimator, but is nearly unbiased.

Table 3 shows the results of scenario 3. Since in this scenario the true parameters change continuously over time, the scenario tests if the adaptive scheme given by the similarity measure separates similar regions from dissimilar regions in an adequate way. The results are analogous to the results of scenario 2—again, the exponentially weighted estimator shows a much larger standard deviation than the similarity weighted estimator. For days 1000 and 2500 the similarity weighted and exponentially weighted estimators also deviate from the theoretical values due to the fast-changing structures. However, they are both much closer to the theoretical value than the unweighted estimator.

It is worth noting that, in all scenarios, the bias of the similarity weighted estimator is similar to the bias of the exponentially weighted estimator. The standard deviation of the similarity weighted estimator, however, is only slightly larger than the standard deviation of the unweighted estimator and much smaller than the standard deviation of the exponentially weighted estimator. The chosen number of 16 assets may seem small from a practical point of view. To ensure the scaling of the results with dimensions, the same study was conducted for 100 assets with analogous correlation structures, leading to equivalent results. The results of the larger study may be obtained from the authors on request.

5. Application to financial data

In this section, we apply our estimator to financial data in the context of mean–variance portfolio allocation. The application is motivated by Engle and Colacito (2006), who showed that the realized volatility of theoretically optimal portfolios is lowest if the covariance matrices for the optimization process are correctly specified. We therefore compare the realized volatility and return of various portfolios drawn from the S&P 500. The study shows that portfolios based on the similarity weighted estimator as discussed in this paper outperform alternative portfolios. We conclude that these similarity weighted estimators perform very well in real data applications.

The value, $V$, of a portfolio consisting of $K$ assets with prices $S_i$ and corresponding portfolio weights $w_i$ ($i = 1, \ldots, K$) is given by

$$V = \sum_{k=1}^{K} w_k S_k = w^T S,$$

(12)

where $S$ refers to the $(K \times 1)$ vector of asset prices and $w$ contains the respective weights.

Consider an investment period from day $t=0$ to day $t=T$. Let $\Sigma$ and $\mu$ be the covariance matrix and the expectation of the $K$ asset returns over the period. Then the portfolio variance and expectation at time $t=T$ are given by

$$\text{Var}(V_T) = w^T \Sigma w,$$

$$E[V_T] = V_0(I + w^T \mu),$$

where $I$ is a vector of ones. Let $\delta V_t$ denote the daily returns of the portfolio over the investment period. Then

$$RV = \sum_{t=0}^{T} (\delta V_t)^2$$

is the realized volatility of the portfolio, which is a measure of the portfolio’s risk over the investment period.

In mean–variance portfolio optimization as introduced by Markowitz (1952), optimal portfolio weights $w_i$ are derived by minimization problems of the form

$$\min_w \left\{ \frac{1}{2} w^T \Sigma w - \gamma w^T \mu \right\},$$

subject to certain constraints, e.g.

$$\sum_{k=1}^{K} w_k = 1$$

(14)

(budget restriction). The parameter $\gamma > 0$ is the investor’s risk tolerance parameter. A value $\gamma = 0$ denotes no risk tolerance. In this case, the investor’s only aim is to minimize the portfolio variance. Large values of $\gamma$ denote risk neutrality, i.e. the investor maximizes the expected portfolio return only.

Since different investors have different risk tolerance levels, we focus on two special cases of the minimization problem. We consider the minimum-variance portfolio (MVP), i.e. the portfolio of minimal variance without further constraints, and the portfolio with minimal variance under the constraint of a fixed target portfolio return $R$ (TRP). The minimum-variance portfolio is the solution of equation (13) when $\gamma$ is set to zero, i.e. the investor is not risk tolerant. To obtain portfolio TRP, $\gamma$ can easily be expressed by the target return $R$

$$\gamma = \frac{R - (\alpha/\beta)}{\mu^T \Sigma^{-1} \mu - (\alpha^2/\beta)}$$

(15)

where $\alpha = 1^T \Sigma^{-1} \mu$ and $\beta = 1^T \Sigma^{-1} 1$. A detailed derivation can be found, for example, in Luenberger (1998).

In a recent paper, Kritzman et al. (2010) argue that minimum-variance portfolios outperform various other
strategies of portfolio optimization, even with respect to their return. By contrast, DeMiguel et al. (2009) raise the question of whether portfolio optimization pays out at all. In their results, optimized portfolios do not significantly outperform naively diversified portfolios, i.e. portfolios where the same amount $1/n$ is invested in $n$ assets. We therefore include this naive portfolio in our study, even though the naive portfolio does not depend on estimators of correlation or covariance. The portfolio strategies MVP and TRP allow us to rank the estimators of the covariance structure according to the portfolio performance, while the outcomes of the naive portfolio confirm the overall plausibility of the results.

The basic idea of the study is to calculate optimal portfolios for every day of our dataset and to evaluate them over some investment horizon $T$ with respect to risk (realized volatility) and return. We then compare the results of the different strategies and estimators. Since mean–variance portfolio optimization requires an inversion of the covariance matrix, the matrices must not be singular. Covariance and correlation matrices based on exponentially weighted estimators are often near to singularity. For example, the standard RiskMetrics approach with $\lambda = 0.94$ results in a time series with an effective length of approximately only 30 trading days. Therefore, exponentially weighted estimators perform poorly in our study and we exclude them from the graphs for clarity and convenience.

We use the same dataset as in section 2, i.e. the 471 assets of the S&P 500 index that are included in the index from 2005 to the beginning of 2010. From this dataset, we randomly choose 10 portfolio constellations of 100 stocks each. For every trading day from August 2007 to November 2009 we compute portfolio weights for the constellations corresponding to the three strategies. The required covariance estimates rely on the similarity weighted estimator and alternatively on the unweighted estimator. For the first estimator, we use a similarity measure, which is determined as discussed in section 2. The probe matrices to calculate the similarity measure rely on moving windows of $L = 50$ trading days and are based on all 471 assets of the dataset. We find that a value of $L = 50$ gives the best results in terms of portfolio risk and stability. Window lengths between 30 and 70 trading days lead to similar results. The results for window lengths around 50 seem to be quite representative for this application. However, other scenarios, markets or applications might require a different window length that needs to be evaluated on different objectives.

The weighting scheme of the estimator includes the $s = 300$ most similar past days. The unweighted estimator is based on a moving window of 300 days. The weights of the target return portfolio also rely on an additionally specified target return $R$ and on estimates of the vector $\mu$ of expected returns. The vector $\mu$ is estimated by the returns of the portfolio’s stocks for every trading day from a moving window of 14 trading days. The target return is then adaptively chosen to be five percentage points above the average entry of $\mu$.

The evaluation results of realized volatility and returns are shown in figures 4 and 5 and in tables 4 and 5. The evaluation periods are 14, 28 and 56 trading days, respectively. The results shown are averages of the 10 portfolio constellations. Visual inspection of the figures shows that the naive portfolio performs worst, especially during the financial crisis. In this time frame, the
incorporation of the covariance structure into the portfolio weights pays off. The realized volatility of the optimized portfolios consistently lies below the realized volatility of the naive portfolios, whereas the similarity weighted scheme obtains the best results. The results are robust for the considered investment horizons, which is shown in tables 4 and 5 in more detail.

In both cases, in the minimum variance portfolio (MVP) as well as in the 5% above market drift portfolio (TRP), the similarity weighting significantly reduces the realized risk. Moreover, the TRP case reveals that the realized return could be improved compared with the unweighted optimization, although the naive portfolio features an even higher return.

6. Conclusion

We have introduced a measure that quantifies the similarity of the correlation structure for two different times. This measure gives a clear indication of drastic changes in the market structure, e.g. at the beginning of the 2008 financial crisis. This measure was adapted to calculate weighted correlation and covariance matrices in which information that originated from a similar market state is weighted higher. We analysed the resulting similarity weighted estimators in a simulation study and applied it to a mean–variance portfolio optimization using empirical data. The results show that our method is able to reduce the portfolio volatility and to enhance the realized return compared with the use of unweighted correlations. The application of similarity weighted estimators is especially advantageous in periods in which the market structure changes drastically.

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