First law applied to flow processes

Derivation of General Energy Equation

The First Law of Thermodynamics was derived for a system, i.e. a fixed collection of matter. But in most engineering problems we would like to focus our attention on a piece of equipment through which material flows continuously, e.g. cylinder of internal combustion engine, the turbine in a power plant, etc.

In order to deal with such a problems we have to resort to a new concept called Control Volume.

Control volume definition:
A control volume is any volume of fixed shape and in a fixed position and orientation relative to the observer. The control volume boundary is called the control surface.

Mass as well as heat and work can cross the control surface and the mass in the control volume as well as the properties of this mass can change with time. The problem is how to apply the First Law, which was developed for a system to a control volume analysis. For this purpose consider a general flow problem as shown below:

This problem is a rate problem, i.e. time dependent. Up to now we have only considered problems which are independent of time. After all time is not a relevant parameter when a system is in equilibrium. This time-dependency provides us with a clue to solve the problem.

Consider a system which is identical to a control volume. Let us consider the change in the system over a finite time interval dt.
We have allowed the flexible system boundary to enclose some matter in the inlet "pipe" at time $t$, and to enclose some other matter in outlet "pipe" at time $t + dt$. Apart from these places, at specified times, the system boundary is identical to the control surface. Thus over a time interval $dt$, $dmi$ mass of fluid enters the control volume and $dme$ leaves the control volume. Heat and work interactions also occur during time $dt$, so by first law:

$$\frac{\delta Q}{\delta t} = \frac{\delta W}{\delta t} + \frac{dE}{\delta t}.$$ 

Let us now consider each of the terms of the first law as it is written for the system and transform each term into an equivalent form that applies to the control volume.

(i) **Energy**

Let $E_t$ = the energy in the control volume at time $t$.

$$E_{t+dt} = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots t + dt.$$ 

Then:

$$E_1 = E_t + ei dmi = \text{Energy of system } dt$$

$$E_2 = E_{t+dt} + ee dme = \text{Energy of system } dt + dt$$

Thus:

$$dE = E_2 - E_1 = (E_{t + dt} - E_t) + (eedme - ei dmi) \quad (2)$$

(i) Change in the total energy inside the control volume in time $dt$.

(ii) The net flow of energy that crosses the control surface during $dt$ as a result of the masses $dmi$ and $dme$ crossing the control surface.

(ii) **Work Done**

$W$ in the equation (1) is not identical with $W_x$ (external work) shown on the diagram. Since displacement work is done by the moving parts of the system boundary.

$$W = W_x - P_i v_i \delta m_i + P_e v_e \delta m_e$$

$$w_{\delta m_i} = \text{Swept volume by different parts of the system Boundary}$$

So $W_x$ is all the work done at the control surface other than that associated with normal forces at places where material crosses the surface e.g. shaft work, shear forces, etc. Thus we can write the energy equation for the control volume as:

$$\frac{\delta Q}{\delta t} + \frac{\delta m_i}{\delta t} (e_i + p_i v_i) = \left( \frac{E_i + \delta t - E_t}{\delta t} \right)$$

$$+ \frac{\delta m_e}{\delta t} (e_e + p_e v_e) + \frac{\delta W_x}{\delta t}$$

But
\[
e + pv = u + pv + \frac{V^2}{2} + gz
\]
\[
= h + \frac{V^2}{2} + gz
\]
\[
\frac{\delta Q}{\delta t} + \frac{\delta m}{\delta t} \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \frac{\delta W}{\delta t}
\]
\[
= \left( E_{i+\Delta t} - E_i \right) + \frac{\delta m e}{\delta t} \left( h_e + \frac{V_e^2}{2} + gz_e \right)
\]

By considering what happens at the limit when \( dt \) tends to 0 and if we include a summation to account for the possibility of additional flow streams entering and leaving the control volume, we get:

\[
Q + \sum m_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) = \frac{dBc \cdot v}{dt} + \sum m_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) + W_x
\]

This is the GENERAL ENERGY EQUATION

It states that the rate of heat transfer into the control volume plus rate of energy flowing in as a result of mass transfer is equal to the rate of change of energy inside the control volume plus rate of energy flowing out as a result of mass transfer plus power output associated with shaft, shear and electrical effects.

**The Steady Flow Energy Equation (SFEE)**

A very large class of devices of interest to engineers such as turbines, compressors, nozzles, boiler and condensers operate under long-term steady-state condition i.e. after the initial start up period they operate in a way that there are no variation of properties with time i.e. All changes in the measurable quantities inside the control volume are non-existent or cyclic. Then \( E_{i+dt} = E_i \) and so

\[
\frac{dE_{i+dt} \cdot v}{dt} \text{ term } \Rightarrow 0. \quad \text{Also } \sum m_i = \sum \dot{m}_e
\]

or there would be an accumulation or fall in masses in the control volume which would nullify the original condition for steady flow. So to obtain steady flow conditions we need to satisfy three conditions:

1. The streams of material crossing the control surface must not change their state or flow rate with time.
2. Each point within the control volume must not change its state with time or only cyclic state variation occur.
3. The heat and work transfer rates must not change with time or the mean rates in the case of cyclic behaviour must not change.

Thus we get the Steady Flow Energy Equation (SFEE)

\[
Q - W_x = \sum_{\text{ext}} m \left( h + \frac{V^2}{2} + gz \right)
\]
\[
- \sum_{\text{in}} m \left( h + \frac{V_i^2}{2} + gz \right)
\]

Or for single streams:

\[
\bar{Q} - W_x = \bar{m} \left( h_2 + \frac{V_2^2}{2} + gz_2 \right) - \left( h_1 + \frac{V_1^2}{2} + gz_1 \right)
\]

**Non-Uniform Streams**

In developing the energy equations we assumed that any given entry or exit stream has a unique velocity. But in all practical cases the velocity at entry is non-uniform and a mean velocity that satisfies the simple continuity equation is used:

\[
\bar{V} = \frac{\bar{m} \cdot \dot{V}}{\rho A} \quad \text{or} \quad \frac{\bar{m}}{A}
\]

**Applications of the SFEE**
(i) **Heat Exchangers**

Consider an evaporator of a refrigeration plant. Liquid Freon enters a coil in contact with the air in the refrigerator cabinet and leaves as vapour.

The velocities are small from SFEE:

\[ \dot{Q} - \dot{W} = \dot{m} \left( h_2 - h_1 \right) + \frac{1}{2} \left( \frac{V_2^2}{m} - \frac{V_1^2}{m} \right) + g \left( z_2 - z_1 \right) \]

- No external work done
- Velocities should be negligible
- Negligible

Thus by finding the change in enthalpy, we find the heat transfer.

(ii) **Adiabatic Nozzles**

A nozzle is used to produce a jet of high velocity by allowing a gas at high pressure and temperature to expand examples in rockets and jet-engines. For small velocity changes the nozzle is convergent:

i.e.
While for high velocity its convergent divergent. Writing SFEE:

\[
\frac{\dot{m}}{\dot{h}} \left( \frac{h_2 - h_1}{m} \right) = \left( h_2 - h_1 \right) + \frac{1}{2} \left( V_2^2 - V_1^2 \right) + g \left( Z_2 - Z_1 \right)
\]

adiabatic \quad \text{No external work being done} \quad \text{can be ignored}

\therefore h_1 - h_2 = \frac{V_2^2}{2} - \frac{V_1^2}{2}

i.e. the increase in k.e. is equal to the decrease in enthalpy.

(iii) Adiabatic Throttling

When a fluid flows through a restriction, such as a half-closed valve in a pipeline, the pressure downstream is always appreciably lower than upstream. This is called throttling, in which one reduces pressure without increasing KE e.g.:

SFEE can be written between (2) and (1) if we choose (2) at a position where outgoing stream is fairly uniform:

\[
\frac{\dot{m}}{\dot{h}} \left( \frac{h_2 - h_1}{m} \right) = \left( h_2 - h_1 \right) + \frac{1}{2} \left( V_2^2 - V_1^2 \right) = 0
\]

adiabatic

But \quad v_2 = v_1

\therefore h_2 = h_1

(iv) Turbine

\[
W_T
\]
The quantity:

\[
\frac{\dot{m}}{\dot{m}} = \frac{\dot{h}_2 - \dot{h}_1}{\dot{m}} + \frac{1}{2} \left( \frac{V_2^2}{V_1^2} - V_1^2 \right) g (z_2 - z_1)
\]

\[V_2 > V_1 \quad \text{negligible}\]

\[
\frac{\dot{m}}{\dot{m}} = \left( h_2 + \frac{1}{2} V_2^2 \right) - \left( h_1 + \frac{1}{2} V_1^2 \right)
\]

\[
\frac{\dot{m}}{\dot{m}} = (h_{02} - h_{01})
\]

\[h_0 = h_2 + \frac{1}{2} V_2^2\]

The quantity is known as the Stagnation enthalpy

**Application of NSFEE (Unsteady flow)**

Consider an initially evacuated tank, fully insulated and with a valve: Now the valve is opened and \( m \) amount of air flows in:

Writing GEE in non-rate form:

\[
\dot{Q} = \dot{m} \Delta E_{cv} + \Delta \left[ m \left( h + \frac{V^2}{2} + gz \right) \right]
\]

\( E_{cv1} = 0 \quad E_{cv2} = m U_f \)

\[\Delta_{out} = 0, \quad \Delta_{in} = mh_m\]

\[m U_f - mh_m = 0 \implies U_f = h_m\]

*System Analysis*
From 1st law:

\[ W = \Delta U + \Delta E \]

\[ W = -P \Delta V \]

\[ \Delta E = m \Delta u \]

\[ \Delta E = m(u_2 - u_1) \]

\[ \Delta E = m(u_2 - u_1) = R \Delta T \]

\[ m = \frac{R \Delta T}{u_2 - u_1} \]

\[ u_2 = u_1 + \frac{R \Delta T}{u_2 - u_1} \]

The container is fully insulated so \( Q = 0 \)

and work done (const P process)

\[ = R \Delta T = -P \Delta V \]

\[ \Delta E = m(u_2 - u_1) \]

\[ \therefore m u_2 - m u_1 = -P \Delta V \]

\[ \therefore m u_2 = m U_1 R \]

\[ \therefore u_2 = h_1 \]

First Law

Thermodynamics